

Dynamic Tides in Close Binaries

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Abstract. The basic theory of dynamic tides in close binaries is reviewed. Particular attention is paid to resonances between dynamic tides and free oscillation modes and to the role of the apsidal-motion rate in probing the internal structure of binary components. The discussed effects are generally applicable to stars across the entire Hertzsprung-Russell diagram, including the binary OB-stars discussed at this meeting.

1. Introduction

In close binaries, each star is distorted by the tidal action exerted by its companion. The tides give rise to a variety of dynamical effects from the resonant excitation of free oscillation modes to the secular evolution of the orbital elements. The effects are richest when the binary is eccentric and the component stars are rotating asynchronously with respect to the orbital motion. The tidal force is then time-dependent and the tides are known as *dynamic tides*. Due to viscous and dissipative effects, the tidal distortion lags behind the position of the companion, creating a torque which tends to circularize the orbit and synchronize the component stars. The time scales of circularization and synchronization depend strongly on the initial binary parameters and on the physical processes responsible for the dissipation of energy. Once circularization and synchronization are achieved, the tidal action is static with respect to a frame of reference corotating with the stars¹. These tides are known as *equilibrium tides*.

The dependence of tidal effects on processes taking place in the stellar interior is a fortuitous circumstance that allows tides to be used to probe physics hidden below the stellar surface. In order to exploit this probing potential, a thorough theoretical understanding is, however, essential. In the following sections, we therefore discuss some basic aspects of the theory of dynamic tides in close binaries. Highlighted applications are the excitation of oscillation modes by resonant dynamic tides and the role of the apsidal-motion rate in studying components of eccentric binaries. We conclude with a brief discussion of future prospects and applications such as the study of the formation of compact objects, the detection of gravitational waves by the NASA/ESA cornerstone mission (*LISA*), and the contribution of large scale photometric variability and exoplanet transit surveys to the study of tides in close binaries.

¹In order for this equilibrium state to set in, the stars' equatorial planes must also coincide with the orbital plane. Throughout this paper, we will always assume this to be the case.

2. The tide-generating potential

We consider a close binary system of stars revolving around each other under the influence of their mutual gravitational force. We denote the orbital period by P_{orb} , the semi-major axis by a , and the eccentricity by e . The first star, with mass M_1 and radius R_1 , is assumed to rotate uniformly with an angular velocity Ω_{rot} perpendicular to the orbital plane. The companion star, with mass M_2 , is treated as a point mass.

The tidal force exerted by the companion is derived from the tide-generating potential $\varepsilon_T W(\vec{r}, t)$, where $\varepsilon_T = (R_1/a)^3 (M_2/M_1)$. In binaries with circular orbits, the parameter ε_T corresponds to the ratio of the tidal force to the gravity at the star's equator, which is approximately equal to the height of the tidal bulge raised on the primary by the secondary.

The tidal distortion of the primary is most conveniently studied by expanding the tide-generating potential in terms of unnormalized spherical harmonics $Y_\ell^m(\theta, \phi)$ and in Fourier series in terms of multiples of the mean motion $n = 2\pi/P_{\text{orb}}$:

$$\varepsilon_T W(\vec{r}, t) = -\varepsilon_T \frac{G M_1}{R_1} \sum_{\ell=2}^4 \sum_{m=-\ell}^{\ell} \sum_{k=-\infty}^{\infty} c_{\ell,m,k} \left(\frac{r}{R_1} \right)^\ell \times Y_\ell^m(\theta, \phi) \exp[i(\sigma_T t - k n \tau)] \quad (1)$$

(e.g. Zahn 1970, Polfiet & Smeyers 1990). Here, G is the Newtonian gravitational constant, (r, θ, ϕ) is a system of spherical coordinates with respect to a frame of reference that is co-rotating with the star, $\sigma_T = k n + m \Omega$ are forcing angular frequencies with respect to the co-rotating frame of reference, and τ is a time of periastron passage. The factors $c_{\ell,m,k}$ are Fourier coefficients defined as

$$c_{\ell,m,k} = \frac{(\ell - |m|)!}{(\ell + |m|)!} P_\ell^{|m|}(0) \left(\frac{R_1}{a} \right)^{\ell-2} \frac{1}{(1 - e^2)^{\ell-1/2}} \times \frac{1}{\pi} \int_0^\pi (1 + e \cos v)^{\ell-1} \cos(k M + m v) dv, \quad (2)$$

where $P_\ell^{|m|}(x)$ is an associated Legendre polynomial of the first kind, and M and v are the mean and true anomaly of the companion in its relative orbit, respectively. The coefficients $c_{\ell,m,k}$ obey property of symmetry $c_{\ell,-m,-k} = c_{\ell,m,k}$ and are equal to zero for odd values of $\ell + |m|$. The coefficients $c_{\ell,m,0}$ are equal to zero for $|m| > \ell - 1$.

The dominant terms in Expansion (1) of the tide-generating potential are the terms associated with $\ell = 2$. The associated Fourier coefficients are independent of R_1/a and are different from zero only for $m = 0$ and $m = \pm 2$. The variations of the coefficients $c_{2,-2,k}$ and $c_{2,0,k}$ as functions of k are shown in Fig. 1 for three different orbital eccentricities e . For a given orbital eccentricity and sufficiently high k -values, the coefficients decrease in absolute value with increasing values of k , but the decrease is slower for higher orbital eccentricities. The number of non-trivially contributing terms in the expansion of the tide-generating potential therefore increases with increasing values of e . For a

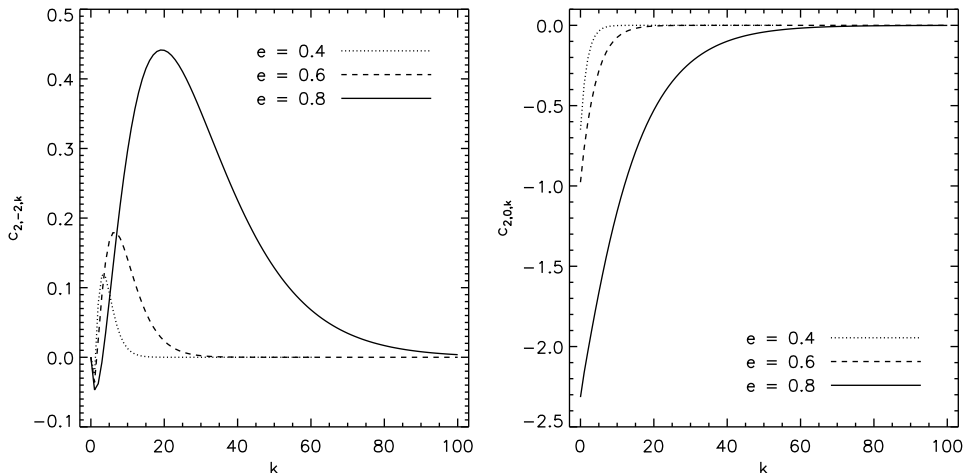


Figure 1. The Fourier coefficients $c_{2,-2,k}$ (left) and $c_{2,0,k}$ (right) as functions of k , for $e = 0.4, 0.6$, and 0.8 . The coefficients $c_{2,2,k}$ tend to be several orders of magnitude smaller than the coefficients $c_{2,-2,k}$ and $c_{2,0,k}$, and are therefore less important in the expansion of the tide-generating potential (see Willems 2003 for details).

given orbital eccentricity, the coefficients $c_{2,-2,k}$ furthermore reach a maximum for k -values for which $kn \approx \Omega_{\text{peri}}$, where Ω_{peri} is the orbital angular velocity of the companion at the periastron of its relative orbit. This is in line with our intuitive expectations that the tidal forcing in eccentric orbits is strongest when the stars pass through the periastron of their relative orbit.

3. Resonant dynamic tides

The Fourier expansion of the tide-generating potential induces an infinite number of forcing frequencies in the tidally distorted star. Non-zero forcing frequencies give rise to dynamic tides, while zero forcing frequencies give rise to static tides. When one of the forcing frequencies is close to the eigenfrequency of a free oscillation mode, a resonance occurs and the mode involved in the resonance is excited by the tidal action exerted by the companion. The possibility of resonances between dynamic tides and free oscillation modes is particularly relevant for the excitation of free oscillation modes g^+ since their eigenfrequencies are most likely to be in the range of the forcing frequencies induced in components of close binaries.

In what follows, we consider the effects of a resonance between a dynamic tide associated with the spherical harmonic $Y_\ell^m(\theta, \phi)$ and the forcing angular frequency $\sigma_T = kn + m\Omega$, and a free oscillation mode of radial order N with eigenfrequency $\sigma_{\ell,N}$ and coefficient of vibrational stability $\kappa_{\ell,N}^2$. At the lowest

²From the orthogonality properties of the spherical harmonics, it follows that tides associated with the spherical harmonic $Y_\ell^m(\theta, \phi)$ can only excite oscillation modes associated with the same $Y_\ell^m(\theta, \phi)$ (see Smeyers et al. 1998 for details).

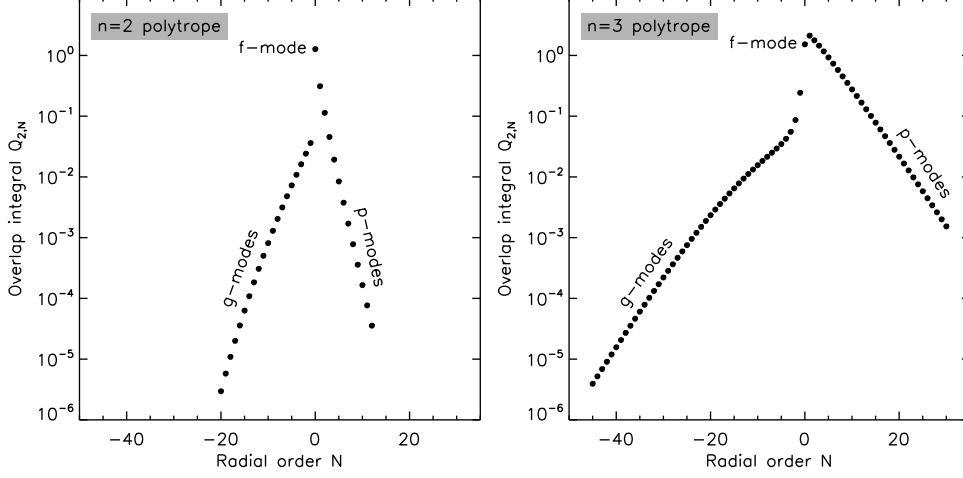


Figure 2. The overlap integrals $Q_{2,N}$ for the polytropic stellar models with indices $n = 2$ (left) and $n = 3$ (right). g^+ -modes are denoted by negative radial orders, p -modes by positive radial orders, and the f -mode by radial order 0. All overlap integrals are determined by normalizing the radial component of the Lagrangian displacement of the resonant modes at the star's surface to unity.

order of approximation, the tide gives rise to a displacement field

$$\vec{\xi}_{\text{res}}(\vec{r}, t) = \frac{\varepsilon_T c_{\ell,m,k}}{2} \frac{\sigma_{\ell,N} Q_{\ell,N} \vec{\xi}_{\ell,N}(\vec{r})}{[(\sigma_{\ell,N} - \sigma_T)^2 + \kappa_{\ell,N}^2]^{1/2}} \exp[i(\sigma_T t - k n \tau + \psi_{\text{res}})], \quad (3)$$

where $\vec{\xi}_{\ell,N}$ is the vector of the Lagrangian displacement of the oscillation mode involved in the resonance,

$$Q_{\ell,N} = \frac{G M_1}{R_1} \frac{\int_{M_1} \vec{\xi}_{\ell,N}(\vec{r}) \cdot \nabla \left[(r/R_1)^\ell Y_\ell^m(\theta, \phi) \right] dm}{\sigma_{\ell,N}^2 \int_{M_1} |\vec{\xi}_{\ell,N}(\vec{r})|^2 dm} \quad (4)$$

is the so-called resonance coefficient (Zahn 1970) or overlap integral (Press & Teukolsky 1977), and

$$\psi_{\text{res}} = -\arctan \frac{\kappa_{\ell,N}}{\sigma_{\ell,N} - \sigma_T} \quad (5)$$

is the phase shift between the tidal displacement field and the resonant term in the expansion of the tide-generating potential (Willems et al. 2003).

The overlap integral $Q_{\ell,N}$ is proportional to the work done by the tidal force through the resonantly excited mode. For main-sequence stars, $Q_{\ell,N}$ generally decreases rapidly with increasing values of N , so that resonant excitation tends to be easier for lower order modes than for higher order modes. The cut-off radial order above which resonant excitation becomes less feasible depends strongly on the internal structure of the star. This is illustrated in Fig. 2 for

the $\ell = 2$ overlap integrals in two polytropic models with different degrees of central condensation. For highly condensed stellar models, the behavior of the overlap integral becomes considerably more erratic than the smooth trends displayed in Fig. 2, although the decrease of $Q_{\ell,N}$ with increasing N is generally conserved for sufficiently high-order modes. Evolved stellar models with sharp transition zones between layers of different chemical compositions however show a considerably more complicated pattern of overlap integrals. Fontaine et al. (2003), for instance, have shown that mode trapping and confinement effects of g^+ -modes in subdwarf B stars, lead to increasing $Q_{\ell,N}$ -values with increasing radial order N . These stars are therefore extremely susceptible to resonances between dynamic tides and free oscillation modes, and are potentially some of the most promising sources to look for forced oscillations in close binaries.

Resonant dynamic tides can furthermore significantly accelerate the secular evolution of the binary's semi-major axis and the star's rotational angular velocity (Savonije & Papaloizou 1983, 1984; Willems et al. 2003). Since the forcing angular frequencies σ_T depend on n and Ω_{rot} , one would therefore expect resonances to be fairly short-lived as the binary evolves rapidly through and away from them. Witte & Savonije (1999, 2001), however, have shown that when stellar and orbital evolution are both taken into account, the changes in the forcing frequencies and the eigenfrequencies may compensate one another, locking the binary in a resonance for a prolonged period of time. Such resonance lockings greatly improve the detectability of resonantly excited oscillation modes through the enhanced variability of the star's surface properties or through the enhanced secular evolution of the binary and its component stars.

Table 1. Examples of stars oscillating with observed frequencies σ_{obs} equal to integer multiples of the orbital frequency n .

Name	M_1	M_2	P_{orb}	e	σ_{obs}/n	Refs.
HD 177863	$3.5M_{\odot}$	$1.0-2.0M_{\odot}$	11.9 d	0.60	10	1, 2, 3
HD 209295	$1.8M_{\odot}$	$0.6-1.5M_{\odot}$	3.11 d	0.35	3, 5, 7, 8, 9	4
HD 77581	$23-29M_{\odot}$	$1.8-2.4M_{\odot}$	8.96 d	0.09	1, 4	5

References: (1) De Cat et al. 2000; (2) De Cat 2001, (3) Willems & Aerts 2002; (4) Handler et al. 2002; (5) Quaintrell et al. 2003.

Despite the possibility of resonance lockings, firm evidence for the presence of resonantly excited oscillation modes in components of close binaries remains scarce. The most promising candidates so far are the multi-periodic oscillators HD 177863, HD 209295, and HD 77581 listed in Table 1. In all three cases, the primary exhibits pulsations with frequencies equal to an integer multiple of the orbital frequency, strongly suggesting the possibility of resonantly excited oscillation modes (note that in the observer's non-rotating frame of reference the condition for a mode with an observed frequency σ_{obs} to be resonantly excited is $\sigma_{\text{obs}} \approx kn$). Additional support for the resonant excitation of these modes is hard to obtain due to the lack of accurately known rotational angular velocities. Willems & Aerts (2002), e.g., have shown that in the case of HD 177863, the

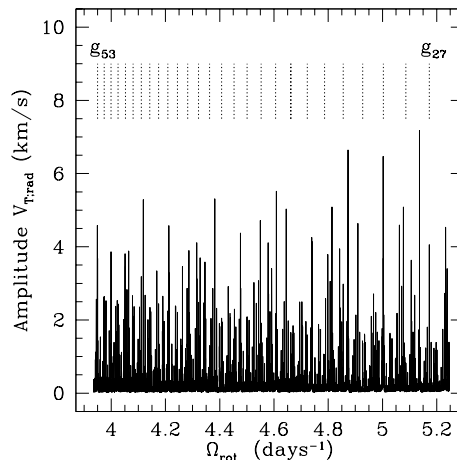


Figure 3. Theoretically predicted amplitude of the $\ell = 2$ tidally induced radial-velocity variations in HD 177863 as a function of the unknown rotational angular velocity Ω_{rot} . The orbital inclination was set to $i = 35^\circ$ and the companion mass to $M_2 = 2 M_\odot$. The dashed vertical lines indicate resonances for which $\sigma_{\text{obs}} \approx 10 n$, corresponding to the observed commensurability of frequencies (see Table 1). Details of the calculation can be found in Willems & Aerts (2002).

errors on the rotational angular velocity introduce uncertainties in the forcing frequencies that are larger than the typical separation between two successive resonances. Theoretically one therefore finds a whole range of possible resonances (see Fig. 3), which severely hinders any definite identification of the resonantly excited mode.

4. Apsidal motion

The tidal distortion of close binary components perturbs the spherical symmetry of the external gravitational field. This perturbation in turn gives rise to time-dependent variations of the orbital elements, the magnitude of which depends on the internal structure of the stars. The motion of the stars in the perturbed gravitational field can therefore be used to probe the interior of close binary components.

Particularly useful for this purpose is the rate of secular change of the longitude of the periastron ϖ . The phenomenon is periodic on time scales of the order of $10\text{--}10^6$ years, and is caused by a combination of the tidal and rotational distortions of the binary components as well as general relativistic effects. Here, we focus on the contribution of tides to the rate of secular apsidal motion. Elaborate discussions and references on the contribution of rotational and general relativistic effects can be found in Giménez (1985), Claret & Giménez (1993), and Claret & Willems (2002).

The most commonly used formula for the rate of secular apsidal motion is due to Cowling (1938) and Sterne (1939). These authors derived the contribution of the tidal distortion to the apsidal-motion rate under the assumption that the orbital and rotational periods are long in comparison to the periods of the free oscillation modes of the component stars. The tidal distortion can then be approximated as static and the resulting apsidal-motion rate due to the dominant $\ell = 2$ tides is given by

$$\left(\frac{d\varpi}{dt}\right)_{\text{tides}} = \left(\frac{R_1}{a}\right)^5 \frac{M_2}{M_1} \frac{2\pi}{P_{\text{orb}}} k_2 15 f(e^2), \quad (6)$$

where

$$f(e^2) = (1 - e^2)^{-5} \left(1 + \frac{3}{2}e^2 + \frac{1}{8}e^4\right). \quad (7)$$

The constant k_2 , which is known as the apsidal-motion constant, depends on the internal structure of the star and measures the extent to which mass is concentrated towards the stellar center. The constant takes the value $k_2 = 0$ in the case of a point mass and the value $k_2 = 0.75$ in the case of the equilibrium sphere with uniform mass density. For main-sequence stars, k_2 is typically of the order of $10^{-3} - 10^{-2}$.

The fairly straightforward dependence on the internal density profile and the independence on less well constrained dissipative effects such as radiative and convective damping are the main ingredients that make the apsidal-motion rate a simple but powerful tool to study stellar physics. Comparisons between theoretically and observationally derived apsidal-motion rates (and thus between theoretical and observational k_2 -values) go back to Schwarzschild (1958) and continue to be updated as new observations and improved stellar models become available (e.g. Claret & Giménez 1993, Claret & Willems 2002). Much of the simplicity of Eq. (6) is, however, related to the assumption that the orbital and rotational periods are long in comparison to the periods of the free oscillation modes of the component stars. When this assumption is violated, dynamical effects become important and the time-dependent response of the star to the tidal forcing must be taken into account (Smeyers & Willems 2001).

The rate of secular apsidal motion accounting for the effects of dynamic tides can be cast in a form similar Eq. (6), provided that the apsidal-motion constant k_2 is replaced by a generalized apsidal-motion constant $k_{2,\text{dyn}}$. The latter depends on the orbital and rotational periods as well as on the orbital eccentricity, and can be interpreted as a weighted mean of the response of the star to the forcing frequencies σ_T (Claret & Willems 2002). The constant $k_{2,\text{dyn}}$ can be negative as well as positive, so that periastron recessions can occur in addition to periastron advances. These periastron recessions, however, usually only arise for close resonances in which the forcing frequency of the resonant dynamic tide is slightly larger than the eigenfrequency of the oscillation mode involved in the resonance.

The effects of dynamic tides on the rate of secular apsidal motion for $10 M_\odot$ and $20 M_\odot$ ZAMS stars are illustrated in Fig. 4 where the relative differences between k_2 and $k_{2,\text{dyn}}$ are displayed as a function of the orbital period for two different orbital eccentricities. The relative differences are mostly negative, so that for binaries with shorter orbital periods Eq. (6) yields somewhat too small

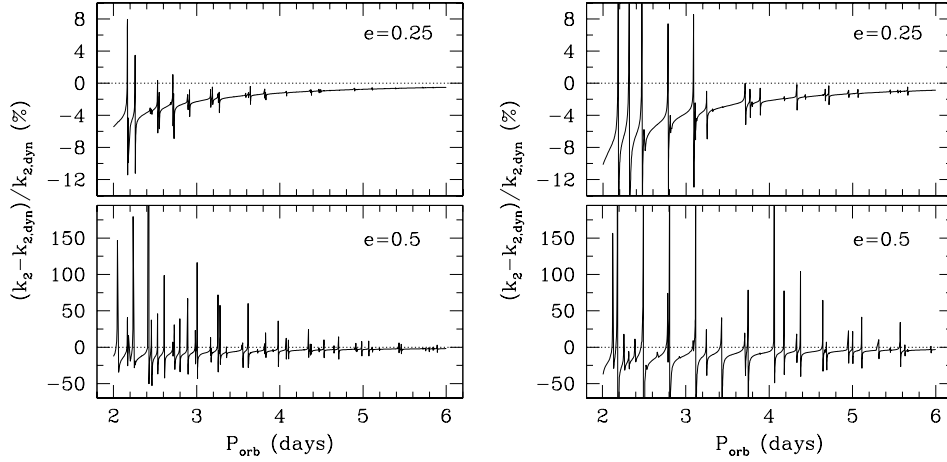


Figure 4. Relative differences between the classical apsidal-motion constant k_2 and the generalized apsidal-motion constant $k_{2,\text{dyn}}$ accounting for the effects of dynamic tides, for $10 M_\odot$ (left) and $20 M_\odot$ (right) ZAMS stellar models and orbital eccentricities $e = 0.25$ and $e = 0.5$. The rotational angular velocity was assumed to be small and set equal to $\Omega_{\text{rot}} = 0.01 n$. Details of the calculation can be found in Smeyers & Willems (2001).

values for the rate of secular apsidal motion, and thus somewhat too long apsidal-motion periods. The peaks observed at shorter orbital periods are caused by resonances of dynamic tides with free oscillation modes g^+ of the tidally distorted star. These peaks are superposed on a basic curve which represents the systematic deviations caused by the increasing role of the stellar compressibility at shorter orbital periods.

The systematic deviations due to the stellar compressibility increase with increasing mass of the ZAMS star. This behaviour is related to the decrease in the amount of central condensation and the associated increase in the impact of tides with increasing stellar mass. The systematic deviations also become larger with increasing orbital eccentricity due to the larger number of higher-frequency tides contributing to the tide-generating potential. The number of resonant dynamic tides on the other hand tends to decrease with increasing stellar mass. The reason for this is that ZAMS stars with higher masses have smaller radiative envelopes which causes their eigenfrequencies to be larger and more widely spaced than those of a lower-mass ZAMS star. For a more extended discussion of the relative differences between k_2 and $k_{2,\text{dyn}}$ for ZAMS stars, we refer to Smeyers & Willems (2001).

The extent of the deviations caused by the compressibility of the stellar fluid also depends on the evolutionary stage of the star (Willems & Claret 2002). The dependency is primarily through the evolution of the radius and the associated change of the star's dynamical time scale. In particular, as the star evolves on the main sequence, the radius and the dynamical time scale increase so that in relative terms the orbital period becomes shorter in comparison to the star's dynamical time scale. Correspondingly, the forcing frequencies become larger when expressed in units of the inverse of the star's dynamical time scale. The

deviations due to the compressibility of the stellar fluid are therefore larger for a model with a larger radius. The evolution of the star furthermore also affects the deviations caused by the resonances of dynamic tides with free oscillation modes of the component stars. In particular, the effects of the resonances tend to be larger for stars near the end of core-hydrogen burning than for stars on the zero-age main sequence. This effect is again related to the size of the radiative envelope, which increases with increasing age of the star on the main sequence.

5. Tides in compact object binaries

A sometimes under appreciated aspect of the study of tides in close binaries is its contribution to understanding the *past* formation and evolution of binaries and their component stars. This is particularly true for binaries containing compact objects such as neutron stars and black holes. These objects are known to be formed in violent supernova explosions or core collapse events during which a large fraction of the system mass is lost and asymmetries in the stellar interior can impart kick velocities of several hundreds of km/s to the compact object at birth. The mass loss and supernova kicks can either expand or shrink the orbit and almost always induce a non-zero eccentricity. X-ray binaries in which a neutron star or black hole accretes matter from a Roche-lobe filling companion, however, typically have nearly circular orbits. Tidal circularization must therefore have been active sometime between the formation of the compact object and the onset of the mass-transfer phase (unless the mass-transfer process itself provides an efficient circularization mechanism). A detailed understanding of tidal effects is then essential if one wishes to reconstruct the evolutionary history of the binary to learn something about the formation of the compact object and its progenitor³. Willems et al. (2005), e.g., showed that for GRO J1655-40, a more accurate quantitative knowledge of tidal effects would tighten the constraints on the progenitor of the central black hole (an OB-star stripped from its hydrogen-rich envelope) by as much as 30%.

Another promising future application involving tidal effects in compact object binaries is the study of gravitational waves emitted by double white dwarfs. These objects originate from binaries initially consisting of two B stars and are expected to be the single most abundant astrophysical sources guaranteed to be detectable by the Laser Interferometer Space Antenna (*LISA*) scheduled to launch in 2015. Double white dwarfs with orbital periods longer than $\simeq 1000$ s are expected to emit gravitational waves with a constant amplitude and a constant frequency equal to twice the orbital frequency. For shorter-period systems, the emission of gravitational waves drains the orbital energy, causing the white dwarfs to spiral in towards each other. So far, all predictions for this inspiral are based on the assumption that both white dwarfs can be treated as point masses. However, as the orbit decays, finite size effects become increasingly important, especially when the orbital frequency sweeps through the spectrum of eigenmode frequencies, and tidal effects inevitably contribute to the orbital inspiral. Since the detection of gravitational waves by laser interferometers is done by template

³Note that such a reconstruction is only possible if the present-day orbital eccentricity is not identically equal to zero.

matching techniques, the inclusion of these effects in predicted gravitational wave signals is imperative to the successful detection of double white dwarfs by *LISA*. In this way, successful detections will also yield a wealth of information on the physics of white dwarfs, and thus the cores of their B-star progenitors, which may prove inaccessible through standard electromagnetic windows on the Universe used by, e.g., asteroseismologists.

6. Future prospects and concluding remarks

Tides in close binaries give rise to a wide range of dynamical effects with vastly different orders of magnitude. Their physical origin can be as simple as the differential gravitational attraction giving rise to the tidal bulge, and as complicated as the convective and radiative damping mechanisms responsible for circularizing the orbit and synchronizing the binary components with their orbital motion. A brief summary of the driving mechanism behind different tidal effects and the associated orders of magnitude is given in Table 2.

Table 2. Summary of the origin and order of magnitude of tidal effects in close binaries.

Tidal Effect	Origin	Order of Magnitude
tidal bulge	differential gravitational attraction	$(R_1/a)^3$
apsidal motion	perturbation of the gravitational field	$(R_1/a)^5$
synchronization	energy dissipation/tidal torque	$(R_1/a)^6$
orbital decay	energy dissipation/tidal torque	$(R_1/a)^8$
circularization	energy dissipation/tidal torque	$(R_1/a)^8$

The variety in the order of magnitude and physical background of the effects listed in Table 2 makes tides a potentially very versatile probe of different regimes of stellar physics. The main difficulty in using tides as asteroseismological probes is currently posed by the quality of the observations. Firstly, the steep dependence of tidal effects on the ratio R_1/a demands very accurate stellar radii and orbital separations in order to get reliable orders of magnitude. Secondly, in binaries that are close enough for dynamical tides to be important, the dependence of the tidal forcing frequencies on the rotational angular velocity requires accurate determinations of the stellar rotation rates in order to properly account for resonant excitations of free oscillation modes.

Any hopes of realizing the required accuracies in the near future most likely rest on vigorous studies of eclipsing binaries in which the eclipse geometry allows precise determinations of the stellar radii and the strongly constrained orbital inclination helps pinning down the components' rotation rates. In view of the spectacular number of ongoing and upcoming large-scale photometric variability and exoplanet transit surveys, the odds of finding eclipsing binaries with fortuitous system parameters are increasingly favorable though. The next decade therefore seems to be an extremely promising era for the exploitation of tides as

probes to study the formation and evolution of binaries with component stars across the entire Hertzsprung-Russell diagram.

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S. Owocki: Could you comment on the role of the tidal forcing effects on cases where the star is rapidly rotating, i.e. near critical rotation?

B. Willems: The study of tides in rapidly rotating stars suffers the same difficulties as the study of free oscillations in rapidly rotating stars. Currently there is no theory available to deal with either of them. Once a theory describing the

effects of rapid rotation on free oscillation modes becomes available, a generalization to tides should be fairly straightforward.

A. T. Okazaki: In a Keplerian circumstellar disk in a highly eccentric system, the particles in the outer parts of the disk have the same frequency as the instantaneous orbital frequency of the companion. Do resonant dynamic tides work on these particles in the outer disk?

B. Willems: Yes, although the problem evidently has a different base geometry than resonances in a spherically symmetric equilibrium star. Lubow (1981), e.g., has shown that resonant accretion disk tides can generate horizontally propagating waves which contribute to the transfer of disk angular momentum to orbital angular momentum.

J. Bjorkman: For a wide, highly eccentric binary with a close periastron passage, there will be a characteristic time scale for the tidal interaction. Is it possible that such an interaction will resonantly excite stellar pulsations for a few e-foldings of their growth? If so, can you describe what conditions would be favorable for this? Such a mechanism might act as a trigger for mass ejections from a near-critically rotating star.

B. Willems: As long as the orbital period is short in comparison to the mode damping or e-folding time, the oscillations will be sustained and “re-excited” during each successive periastron passage. The conditions for resonant excitation become increasingly favorable with decreasing orbital period and increasing orbital eccentricity (i.e. with decreasing periastron passage time).

G. Meynet: The deformation of the star due to tidal forces might induce some kind of meridional circulation. Do you know if this process is important? Could it induce some kind of mixing of the elements?

B. Willems: I am not aware of any work looking into tidally induced meridional circulation. It is conceivable, however, that the tidal distortion creates a temperature difference between the poles and the equator similar to that due to rotational flattening. Whether or not any resulting meridional circulation penetrates deep enough into the star to mix the elements remains to be seen.